Part D.

1. Let A'' be an intersection point of $F'A_1$ and B_1C_1. Points B'' and C'' are defined similarly. Prove that triangle $A''B''C''$ is autopolar wrt the incircle and the vertices of triangle ABC lie on their sides.
2. Prove that the points which are symmetrical to points A and C wrt A_1 and C_1 respectively lie on line $A''C''$.
3. Prove that BC_1A_0C'' and BA_1C_0A'' are parallelograms.
4. Prove that $F' = F$. I.e. F' is the Feuerbach point of triangle ABC.
5. Let a be a length of BC, b be a length of AC and c be a length of AB. Prove that $(a+c-b)^2 = ac$.
6. Prove that if $(a+c-b)^2 = ac$ then our condition (about point A, G, I and C) is true.
7. Prove that F is a centroid of triangle $A'B_0C'$.
8. Let S be midpoint of $A'C'$. Prove that FB_0 pass through it.
9. Prove that S is a midpoint of segment BO_B.
10. Prove that SE is parallel to BB_1.
11. Prove that O_BF is parallel to BB_1.
12. Prove that F is centroid of triangle $O_BA_0C_0$.
13. ($\forall$)Let L_B be a pole of A_0C_0 wrt the incircle. Prove that L_B is intersection point of $A'C'$ and GE.
14. ($\forall$)Prove that I is orthocenter of triangle ACL_B.
15. ($\forall$)Let G' be a point isogonal conjucated to point G. Prove that G' is center of homotety of the incircle and the circumcircle of the initial triangle.
16. Prove that G' is symmetrical to point G wrt bisector of angle $\angle B$.
17. ($\forall$)Prove that points F, G' and M are collinear.
18. Prove that FM is parallel to bisector of angle $\angle B$.
19. Prove that GM is parallel to $A'C'$.
20. Prove that O_B is midpoint of BL_B.
21. Prove that F is a centroid of triangle $A_1L_BC_1$.
22. Prove that the common point of A_1C_1 and OI is isogonaly conjugated to L_B.

Part X.

1. (∀)Prove that the image of a line which not pass through the vertices of the triangle under isogonal conjugation is a conic.

2. (∀)Prove that the image of line OI under isogonal conjugation is a conic which tangents this line.

3. (∀)Prove that the image of line OI under isogonal conjugation is a hyperbola with perpendicular asymptotes. This hyperbola is called Feuerbach hyperbola.

4. (∀)Prove that points A, B, C, I, H, G and N lie on Feuerbach hyperbola where N is a Nagel point of triangle ABC.

5. (∀)Let l be an arbitrary line passing through O. Denote by l' conic which is an image of line l under isogonal conjugation. Prove that center of l' lie on Euler circle.

6. (∀)Let an equilateral hyperbola pass through the vertices of the triangle ABC and P be an arbitrary point on it. Prove that the circumcircle of the pedal triangle of point P passes through the center of this hyperbola.

7. (∀)Prove that center of the Feuerbach hyperbola is a Feuerbach point.

8. (∀)Prove that L_B lies on the Feuerbach hyperbola.

9. (∀)Prove that line FB_1, the circumcircle of triangle A_1C_1I and Feuerbach hyperbola have two common points.

10. Prove that locus of centers of images of lines passing through I under isogonal conjugation is an ellips which passes through the feet of bisectors, the midpoints of the sides and the Feuerbach point.

11. Prove that lines B_0L_B and GM intersect on the Feuerbach hyperbola.

12. (∀)Prove that intersection point of lines FB_0 and A_1C_1 is a pole of line AC wrt the Feuerbach hyperbola.

13. (∀)Prove that the pole of line $A'C'$ wrt the Feuerbach hyperbola is a point which is the reflection of B_1 in E.

Part F.

1. Let Q'' be a reflection point of Q in Q'. Point P'' is defined simultaneously. Prove that $P''Q'' = AC$.
2. Prove that in triangle $P''FQ''$:
   - FB is a median.
   - FI is an altitude.
   - FG' is a line joining F with the circumcircle of triangle $P''Q''F$.
   - FG is a symmedian.
3. Prove that line KB_1 passes through the intersection points of $A'E$ with BC and $C'E$ with AB.
4. Prove that line KB_1 passes through the intersection point of $A'C'$ and A_0C_0.
5. Prove that GG' and BB' intersect on the Feuerbach hyperbola.
6. Prove that FG' and BB' intersect on the circumcircle of triangle ABC. Denote this point D.
7. Prove that line L_BB_0 passes through D.
8. Denote the Feuerbach point of triangle $BP'Q'$ by F''. Prove that FF'' is parallel to BB_1.
9. Prove that lines FO_B, EB_0 and BB' are concurrent.
10. Prove that the Feuerbach hyperbola of triangle $P''Q''B$ passes through point B'.
11. Prove that the Feuerbach hyperbola of triangle $P''Q''B$ passes through L_B.
12. Let C_1' be a touching point of incircle of triangle $BP''Q''$ with side QB. Points A_1' and B_1' are defined simultaneously. Prove that the image of line $A_1'C_1'$ under homothety with center F and coefficient 4 is line A_1C_1.
13. ($\forall$)Prove that lines A_0A', B_0B', C_0C' and the tangent to the incircle through F are concurrent. Denote this intersection point by M'.
14. Prove that M' lies on the bisector of angle $\angle B$.
15. ($\forall$)We intersect lines passing through F and parallel to AA_1, BB_1, CC_1 with lines AA', BB', CC' respectively. Prove that these three points lie on line GM.
16. ($\forall$)We intersect lines passing through M' and parallel to AA_0, BB_0, CC_0 with lines AA', BB', CC' respectively. Prove that these three points lie on line IM.