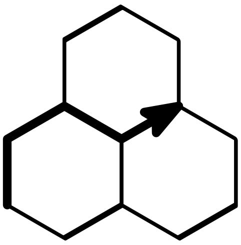

Pavements, colorings and tiling groups

♦ **F1.** Consider an oriented graph whose edges are colored in two colors, and each vertex is the end of one red and one blue edge and the origin of similar edges. A self-coincidence of the graph is a rule which maps each vertex of the graph into some vertex so that each vertex has a single inverse image. Suppose that for each pair of vertices there exists a self-coincidence of the graph which saves orientation and colors of edges and maps the first vertex to the second one. If we pass a path such that first three edges are red and the remaining three edges are blue then we come to the same point as if first three edges were blue and the remaining three edges were red. Prove that the result remains true if 3 is replaced by 24.

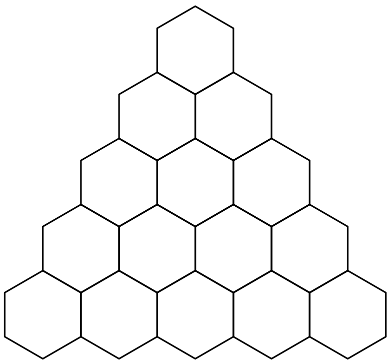

Figure 1. T_5

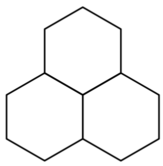

Figure 2. T_2

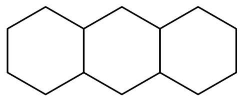

Figure 3. L_3

Define the domain T_n as a "triangle" formed of hexagons. Furthermore define L_n as n hexagon placed in a row (see fig. 1–3)

♦ **F2.** Find a correspondence between figures on a hexagon lattice and tiles on a square lattice. Place arrows of two colors (a and b) in the square lattice so that words corresponding to tiles L_n produce closed paths.

♦ **F3.** Construct a coloring (invariant) of a new square lattice from $F2$ such that paths L_4 correspond to zero values and figures T_n do not.

♦ **F4.** Prove that T_n cannot be tiled by figures L_3.

♦ **F5.** Find all values n such that T_n can be tiled by figures T_2.