

48th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2026

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

points problems

- 4 1. Alice has a set of 25 weights, with masses $a_1, \dots, a_{25}$; Bob also has a set of 25 weights, with masses $a_1 + a_2, a_2 + a_3, \dots, a_{25} + a_1$. Bob was able to put all his weights on the two pans of a balance scale so that the pans are balanced. Prove that Alice can add some of her weights (at least one) to each pan so that the pans are balanced again.

Pavel Kozhevnikov

- 4 2. The diagonals of parallelogram $ABCD$ intersect at point O . A point M is chosen on the side BC so that $AM = DM$. Prove that the length of segment OM is equal to half of one of the sides of the parallelogram.

Maxim Volkevich

- 2 3. Alice computed the sum of some n consecutive positive integers, and Bob computed the sum of the next n consecutive integers (immediately following Alice's). Can Bob's sum be divisible by Alice's sum if
- 2 a) $n = 2025$;
- 2 b) $n = 2026$?

Pavel Kozhevnikov

- 5 4. Two players take turns placing dominoes (2×1 rectangles) on a 5×10 grid (5 rows, 10 columns). The first player always places dominoes vertically, and the second player always places them horizontally. The dominoes must follow the grid lines and cannot overlap. The player who cannot make a move loses. Which player can guarantee a win, no matter how the opponent plays?

Ludmila Smirnova

- 3 5. a) Several boxes, each of weight at most 1 ton, were delivered by two boats, each with a capacity of 12 tons. Prove that these boxes can be transferred to 7 trucks, each with a capacity of 4 tons.
- 3 b) Several boxes, each of weight at most 1 ton, were delivered by 7 trucks, each with a capacity of 4 tons. We need to load these boxes onto two boats, each with a capacity of W tons. What is the smallest W for which this is always possible? (W need not be an integer.)

Konstantin Knop

48th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior O-Level Paper, Fall 2026

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points problems

- 4 1. A set of sticks of lengths 100 cm, 101 cm, $\dots$, 200 cm (one stick of each length) is placed on the table. On each move, Alice takes a stick, breaks it into two integer length pieces, and places them back on the table; however, it is required that at all times, any three sticks on the table can form a non-degenerate triangle. Alice continues breaking the sticks as long as it is possible. How many sticks can there be on the table at the end? (Find all possible answers.)

Ivan Russkikh

- 4 2. Point M is chosen inside the parallelogram $ABCD$ so that $\angle AMB = 90^\circ$. The circumcircles of triangles BMC and AMD are tangent to each other. Prove that these circles are equal.

Mikhail Malkin

- 5 3. A circle is divided into 100 sectors, numbered 1 to 100 clockwise; some are black and the rest are white. A painting robot travels around the circle clockwise, starting in sector 1 and finishing in sector 100. Upon arriving at a sector, it looks at the two preceding sectors (that is, when in sector 1, it looks at sectors 100 and 99; when in sector 2, it looks at sectors 1 and 100; and so on). If this two sectors are the same color, it leaves the current sector unchanged; otherwise, it reverses the color of the current sector.
It is known that sector 100 was initially black and remained black after the robot completed its work. What color would sector 100 be at the end if it had initially been white (with all other sectors colored as before)?

Ludmila Smirnova

- 5 4. We are given two infinite increasing progressions of positive integers, an arithmetic progression and a geometric progression. It is known that they have at least two common terms. Is it true that their common terms form an infinite geometric progression?

Boris Frenkin

- 5 5. Two players take turns placing dominoes (2×1 rectangles) on a 10×100 grid (10 rows, 100 columns). The first player always places dominoes vertically, and the second player always places them horizontally. The dominoes must follow the grid lines and cannot overlap. The player who cannot make a move loses. Which player can guarantee a win, no matter how the opponent plays?

Ludmila Smirnova