

points problems

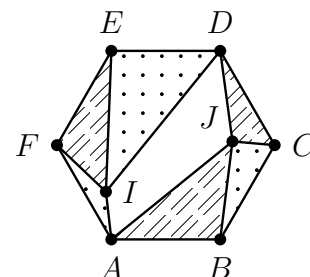
- 5 1. Two concentric circles form a ring of width 1 cm. Is it possible that a rectangle of area at least 1000 cm^2 can be cut out of this ring?

Dmitry Mukhin

- 6 2. Does there exist an integer $a > 1$ such that some rectangle cannot be cut into $1 \times a$ strips, but can be cut into $1 \times (a - 1)$ and $1 \times (a + 1)$ strips so that there are equally many strips of each of these two sizes?

Alexandr Gribalko

- 6 3. Points I and J are chosen inside a regular hexagon $ABCDEF$ so that $AIDJ$ is a convex quadrilateral. The parts of the hexagon outside this quadrilateral are divided into triangles; three of them are shaded with lines, and the other three with dots (see the figure). Prove that the total area of the first three triangles is equal to the total area of the other three triangles.

Mekhrin Mustafadze, Dzhamal Kasymov

- 7 4. Do there exist 100 distinct positive integers such that both their sum and their product are 100th powers of positive integers?

Alexandr Gribalko

- 8 5. Two sages are about to face a test. There are many red, yellow, and green hats. Each sage will have a hat put on their head; each of them will see the other's hat but not their own. At noon, the sages will be taken to two isolated rooms; each room has a clock and three buttons: red, yellow, and green. Each sage presses buttons one at a time, and the two of them together may make at most two presses within one hour. If under these conditions at least one sage presses the button of the colour of their own hat, the test immediately ends, and both sages win. Otherwise, after one hour both sages lose. Can the sages agree on a strategy in advance so that they are guaranteed to pass the test?

Ivan Mitrofanov

- 9 6. A flat island has the shape of a disk. From time to time, a pirate ship arrives at the island, and the pirates spread out over the island to rest. After the captain sees the position of each pirate, he chooses a location for the headquarters. Then the ship sails away, another ship arrives, and so on. Prove that, regardless of how many pirates are on each ship and where they settle on the island, captains can agree in advance to choose location of the headquarters so that the average distance from the headquarters to the pirates will be the same for all ships.

Anton Petrunin

- 9 7. There are 333 identical-looking coins laid out in a row at equal intervals. Three of them are counterfeit: they all have the same weight and are lighter than the genuine coins (which also all have the same weight). Moreover, one of the counterfeit coins is at an equal distance from the other two counterfeit coins. Knowing this information, Baron Munchausen placed several coins on an empty balance scale (the same number of coins on each pan). Could it happen that Munchausen uniquely determined all the counterfeit coins from the result of this single weighing?

Alexandr Gribalko

48th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior A-Level Paper, Fall 2026

(The result is computed from the three problems with the highest scores.)

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- 4 1. Two concentric circles form a ring of width 1 cm. Is it possible that a rectangle of area at least 1000 cm^2 can be cut out of this ring?

Dmirty Mukhin

- 6 2. Alice has an $n \times 2n$ board divided into unit cells, and Bob has a $1 \times k$ strip divided into unit cells. Bob cut his strip along the grid lines into several pieces, each consisting of at most $2n$ cells. What is the largest k for which Alice can always place all these pieces on her board, following the grid lines and without overlaps?

Ivan Chan Kai Chin

- 7 3. A convex quadrilateral $ABCD$ has equal diagonals. Similar isosceles triangles ABP and CDQ are constructed externally on the sides AB and CD as bases, and similar isosceles triangles ADR and BCS are constructed externally on the sides AD and BC as bases. Find the angle between the lines PQ and RS .

Alexey Zaslavsky

- 7 4. We are given a set of $2n + k$ cards, each with a positive integer written on it. It is known that the sum of the numbers on any $n + k$ of the cards is divisible by the sum of the numbers on the remaining n cards. Find the largest possible value of n in terms of k .

Konstantin Filippov

- 10 5. For which n is it possible to colour all the sides and diagonals of a regular n -gon in several colours so that each colour is used for exactly three segments, and the three segments of each colour always form an isosceles triangle?

Alexandr Gribalko

- 12 6. There is a flat island in the ocean. When a pirate ship arrives at the island, the pirates spread out over the island to rest. A collector has obtained several identical maps of this island from different ships; each map shows where each pirate of that ship rested. The collector decided to mark a pirate “headquarters” on each map (a separate one for each map) so that the average distance from the headquarters to the pirates is the same for all maps. Prove that this can be done, regardless of how many pirates were on each ship and where they settled on the island. Assume that the distance between two points is the length of the shortest path between them within the island, and that such a path exists for any two points of the island.

Anton Petrunin

- 14 7. Do there exist 100 distinct positive integers such that both their sum and their product are 60th powers of positive integers?

Ilya Bogdanov, Alexandr Gribalko