

47th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2025

(The result is computed from the three problems with the highest scores.)

points problems

- 4 1. In a class, every student either lies or tells the truth depending on the day of the week (possibly different days for different students), and no student tells the truth for two consecutive days. On April 1st, 2nd, 3rd, and 4th, every student was asked whether he will be telling the truth tomorrow. On April 1st, all students responded “yes”; on April 2nd, half of the students responded “yes”; on April 3rd, one third of the students. What fraction of the students told the truth on April 4th?

Georgy Karavaev

- 4 2. The path from place A to B follows first a paved road and then a forest trail. Alice and Bob started walking towards each other at the same time from places A and B respectively and met on the paved road. Next day, they also started towards each other at the same time, but now Alice started from B , and Bob – from A , and they again met on the paved road, 6 kilometers away from the place of their previous meeting. It is known that each of them walks on the paved road at constant speed, but Alice walks 1.5 times faster than Bob. It is also known that on the forest trail, each of them walks slower than on the paved road, but Alice still walks 1.5 times faster than Bob.

How far their meeting point on the first day was from A ?

Ludmila Smirnova

- 5 3. A rectangular board, consisting of 1×1 squares, is covered by two layers of 1×2 tiles (each tile follows the grid lines of the board; each of two layers completely covers the board). Let us call a top layer tile *special* if its two halves are on top of two different tiles of the bottom layer, with different orientations (i.e., one horizontal and the other one vertical). Is the number of special tiles necessarily even?

Tatiana Kazitsyna, Boris Frenkin

- 5 4. Baron Munchausen has colored every positive integer number using one of three colors, red, white, or blue (all colors were used). He claims that the sum of any 99 red numbers is blue, and the sum of any 99 blue numbers is red (the same number can appear more than once in the sum). Can it be that he is telling the truth?

Mikhail Evdokimov

- 6 5. Does there exist an acute angle α such that some rectangle can be cut into isosceles trapezoids, each of them having α as one of its angles?

Egor Bakaev

47th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior O-Level Paper, Fall 2025

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

points problems

1. We are given a row of boxes labeled $1, 2, 3, \dots$ (left to right). We place numbers $1, 2, \dots, 2025$ (in this order) in these boxes following these rules:
- Each two numbers in the same box must be relatively prime.
 - Each next number is placed in the leftmost box allowed by the previous rule.
- After all numbers are placed, we count how many numbers are in each box.
- 2 a) How many numbers are in box 2?
- 2 b) How many numbers are in box 3?

Alexandr Shapovalov

2. In a convex pentagon $ABCDE$, sides AE , BC , and DE are equal; also, angles A , B , C , and D are all equal. Prove that points A , B , C , D , and E lie on a circle.

Mikhail Evdokimov

3. 30 boys and 30 girls stand in a circle. Prove that it is possible to choose 10 boys and 10 girls such that no two of the chosen people stand next to each other.

Alexandr Gribalko

4. Alice and Bob are drawing on the board. Alice has drawn several circles with different centers, coloring each of the center points either red or blue. It so happened that if any two of these circles are tangent to each other, then they are externally tangent, and their centers have different colors. Is it true that Bob can always replace each circle by a different one with the same center so that the following condition holds: if two old circles were tangent to each other, then the corresponding new circles are also tangent, but now they are internally tangent?

Mikhail Svyatlovsky

5. An $n \times n$ table is filled with integers 0 to n so that in each row and in each column all entries are different. Let us call a table cell *lucky* if the union of its row and column contains all numbers 0 to n .
- 3 a) What is the maximal possible number of lucky cells?
- 3 b) Prove that the number of lucky cells is even.

Alexandr Gribalko