

47th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior A-Level Paper, Fall 2025

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

points	problems	
	1.	In a 12-digit number A , any two adjacent digits form a 2-digit prime number, and all eleven of these primes are distinct.
2	a)	What is the last digit of A ?
2	b)	Give an example of such a number A .
		<i>Georgy Karavaev</i>
6	2.	Angles B and C of a quadrilateral $ABCD$ are both equal to 120° . There is a point on side AD such that from this point, all other sides are seen at the same angle. Find the angle between the diagonals of the quadrilateral.
		<i>Mikhail Evdokimov, Alexey Zaslavsky</i>
	3.	Alice and Bob are playing on a 100×100 square table, each cell of which contains either a plus sign or a minus sign. They take turns crossing out rows and columns of the table: Alice on her turn can cross out any remaining row of the table; similarly, Bob on his turn can cross out any remaining column. They repeat that until only one cell remains. If that cell contains the plus sign, Alice wins; if the minus sign, Bob wins. Does there exist a table such that, no matter if Alice or Bob moves first, there is a winning strategy for
2	a)	the second player
5	b)	the first player?
		<i>Boris Frenkin, Alexey Zaslavsky</i>
	4.	
	a)	Once upon a time, in a far away land there were n cities but no roads between them. The hero started building the roads one by one. Each time he selects two closest cities not connected directly by a straight road and builds a straight road between them. His work ends when it becomes possible to go from any city to any other city (possibly with intermediate stops in other cities). Is it true that no two of the roads he builds will intersect outside of the cities?
2		
6	b)	Same question, but now each time the hero selects the two closest cities which are not connected by roads (even indirectly, with intermediate stops at other cities).
		<i>Alexey Zaslavsky</i>
10	5.	Prove that there exists a positive integer N such that there are exactly 1000 squares strictly between N^3 and $(N + 1)^3$.
		<i>Alexey Tolpygo</i>
	6.	Alice has 60 cards, numbered 1 through 60; each card contains a real number. Bob doesn't know these numbers, but can choose any 17 numbers between 1 and 60 and ask Alice for the sum of the real numbers written on these cards. Can Bob determine the sum of the real numbers on all 60 cards by asking
3	a)	at most 30 questions;
4	b)	at most 20 questions;
5	c)	at most 10 questions?
		<i>Alexey Tolpygo</i>
12	7.	We are given a positive integer k . Someone places n coins of weights $1, 2, \dots, n$ in a circle on the table; all coins look identical. It is known that the weights go in increasing order, but we do not know the starting point and whether they increase clockwise or counterclockwise. You have a balance scale that allow you to take any two coins and find out which one is heavier (you can only place one coin on each side of the scale). Baron Munchausen claims that you can make at most k such weighings and know for certain the weight of at least one coin. What is the largest n for which it is true?
		<i>Ivan Mitrofanov</i>

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Senior A-Level Paper, Fall 2025

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points problems

- 1
- 4
1.
- a)
- b)
- the second player
- the first player?

Boris Frenkin, Alexey Zalsavsky

- 6
2.
- In a convex quadrilateral, we draw a segment through the midpoint of each diagonal parallel to the other diagonal, with endpoints on the sides of the quadrilateral. Show that the endpoints of these two segments form a quadrilateral that has a pair of parallel sides.

Sergey Tokarev

- 4
- 4
3.
- a)
- b)
- $N = 47$;
- $N = 48$?
- N prisoners are held in cells placed in a circle. Tomorrow each prisoner will roll a die; after that, he would make a guess, predicting the numbers rolled by each of the 5 prisoners in cells clockwise from his own. If at least one prisoner guesses at least one of the numbers correctly, all prisoners will be freed. Today, the prisoners can talk and decide on the common strategy, but after midnight, no communication will be possible. How should they act to guarantee their release if

Tatiana Kazitsyna

- 8
4.
- Prove that there exists a positive integer N such that there are exactly 1000 squares strictly between N^3 and $(N + 1)^3$.

Alexey Tolpygo

- 6
- 3
5.
- a)
- b)
- $N = 3$;
- $N = 5$?
- Each side of a regular N -gon contains a robot. Each robot is moving along the side with constant speed that may be different for different robots; upon reaching the vertex, it turns around instantly and continues moving along the same side in the opposite direction. When two robots meet at a vertex, they produce a spark. Can it happen that at each vertex, the sparks appear with the same non-zero frequency if

Alexandr Yuran

- 10
6.
- A snail is crawling on the plane, starting and ending at the same point. Its path consists of n straight segments, without self-intersections. It is known that the snail can only move in three directions: up, right and down-left (at 45° angle to the horizontal). Prove that n is odd. (Adjacent segments cannot have the same direction.)

Pavel Kozhevnikov

- 14
7.
- We are given a positive integer k . Someone places n coins of weights $1, 2, \dots, n$ in a circle on the table; all coins look identical. It is known that the weights go in increasing order, but we do not know the starting point and whether they increase clockwise or counterclockwise. You have a balance scale that allow you to place any number of coins on each of the two sides of the scale and determine which side is heavier. Baron Munchausen claims that you can make at most k such weighings and know for certain the weight of at least one coin. What is the largest n for which it is true?

Alexandr Shapovalov