45th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2023

(The result is computed from the three problems with the highest scores.)

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points	problems
4	1. A strip for playing "hopscotch" consists of ten squares numbered consecutively $1, 2, \ldots, 10$. Clarissa and Marissa start from the center of the first square, jump 9 times to the centers of the other squares so that they visit each square once, and end up at the tenth square. (Jumps forward and backward are allowed.) Each jump of Clarissa was for the same distance as the corresponding jump of Marissa. Does this mean that they both visited the squares in the same order? *Alexey Tolpygo*
4	2. The quadrilateral $ABCD$ is convex. Its sides AB and CD are parallel. It is known that the angles DAC and ABD are equal. Furthermore the angles CAB and DBC are equal. Is $ABCD$ necessarily a square? *Alexandr Terteryan*
5	3. Eight farmers have a checkered 8×8 field. There is a fence along the boundary of the field. The entire field is completely covered with berries (there is a berry in every point of the field, except the points of the fence). The farmers divided the field along the grid lines in 8 plots of equal area (every plot is a polygon), however they did not demarcate their boundaries. Each farmer takes care of berries only inside his own plot (not on its boundaries). A farmer will notice a loss only if at least two berries disappeared inside his plot. There is a crow which knows all of the above, except the location of boundaries of plots. Can the crow carry off 9 berries from the field so that for sure no farmer will notice this? *Tatiana Kazitsyna*
5	4. There are several (at least two) positive integers written along the circle. For any two neighboring integers one is either twice as big as the other or five times as big as the other. Can the sum of all these integers equal 2023? *Sergey Dvoryaninov*
5	5. Alice and Bob have found 100 bricks of the same size, 50 white and 50 black. They came up with the following game. A tower will mean one or several bricks standing on top of one another. At the start of the game all bricks lie separately, so there are 100 towers. In a single turn a player must put one of the towers on top of another tower (no flipping towers allowed) so that the resulting tower has no same-colored bricks next to each other. The players make moves in turns, Alice starts first. The one unable to make the next move loses the game. Who can guarantee the win regardless of the opponent's strategy? *Nikolay Chernyatiev*

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Senior O-Level Paper, Fall 2023

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points	problems
3	1. Baron Munchhausen was told that some polynomial $P(x) = a_n x^n + \ldots + a_1 x + a_0$ is such that $P(x) + P(-x)$ has exactly 45 distinct real roots. Baron doesn't know the value of n. Nevertheless he claims that he can determine one of the coefficients $a_n, \ldots, a_1, a_0$ (indicating its position and value). Isn't Baron mistaken? *Boris Frenkin*
4	2. There are three hands on a clock. Each of them rotates in a normal direction at some non-zero speed, which can be wrong. In the morning the long and the short hands coincided. Just in three hours after that moment the long and the mid-length hands coincided. After next four hours the short and the mid-length hands coincided. Will it necessarily occur that all three hands will coincide? *Alexandr Yuran*
4	3. Consider all 100-digit positive integers such that each decimal digit of these equals 2, 3, 4, 5, 6, or 7. How many of these integers are divisible by 2^{100}? *Pavel Kozhevnikov*
5	4. Given is an acute-angled triangle ABC, H is its orthocenter. Let P be an arbitrary point inside (and not on the sides) of the triangle ABC that belongs to the circumcircle of the triangle ABH. Let A', B', C' be projections of point P to the lines BC, CA, AB. Prove that the circumcircle of the triangle $A'B'C'$ passes through the midpoint of segment CP. *Alexey Zaslavsky*
6	5. Nine farmers have a checkered 9×9 field. There is a fence along the boundary of the field. The entire field is completely covered with berries (there is a berry in every point of the field, except the points of the fence). The farmers divided the field along the grid lines in 9 plots of equal area (every plot is a polygon), however they did not demarcate their boundaries. Each farmer takes care of berries only inside his own plot (not on its boundaries). A farmer will notice a loss only if at least two berries disappeared inside his plot. There is a crow which knows all of the above, except the location of boundaries of plots. Can the crow carry off 8 berries from the field so that for sure no farmer will notice this? *Tatiana Kazitsina*