44th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2022

(The result is computed from the three problems with the highest scores.)

points	problems
3	1. Is it possible to arrange 36 distinct numbers in the cells of a 6×6 table, so that in each 1×5 rectangle (both vertical and horizontal) the sum of the numbers equals 2022 or 2023? *Egor Bakaev*
4	2. Does there exist a natural number that can be represented as the product of two numeric palindromes in more than 100 ways? (A numeric palindrome is a natural number that is read the same from left to right as from right to left.) *Egor Bakaev*
5	3. A pentagon $ABCDE$ is circumscribed about a circle. The angles at the vertices A, C and E of the pentagon are equal to $100°$. Find the measure of the angle ACE. *Mikhail Evdokimov*
5	4. Is it possible to color all integers greater than 1 in three colors (each integer in one color, all three colors must be used) so that the color of the product of any two differently colored numbers is different from the color of each of the factors? *Mikhail Evdokimov*
5	5. Alice has 8 coins. She knows for sure only that 7 of these coins are genuine and weigh the same, while the remaining one is counterfeit and is either heavier or lighter than any of the other 7. Bob has a balance scale. The scale shows which plate is heavier but does not show by how much. For each measurement, Alice pays Bob beforehand a fee of one coin. If a genuine coin has been paid, Bob tells Alice the correct weighing outcome, but if a counterfeit coin has been paid, he gives a random answer. Alice wants to identify 5 genuine coins and not to give any of these coins to Bob. Can Alice achieve this result for sure? *Alexandr Gribalko, Alexey Zaslavsky*

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Senior O-Level Paper, Fall 2022

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

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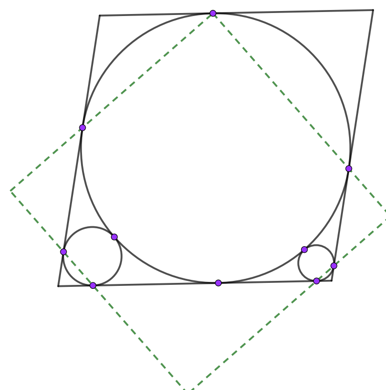
points problems

3 — 1. Find the maximum integer m such that $m! \cdot 2022!$ is a factorial of an integer.

Boris Frenkin

5 — 2. A big circle is inscribed in a rhombus, each of two smaller circles touches two sides of the rhombus and the big circle as shown in the Figure on the right. Prove that the four dashed lines spanning the points where the circles touch the rhombus as shown in the Figure make up a square.

Egor Bakaev



5 — 3. 2022 points are marked on a straight points are the same distance apart. H in red and the other half are colo the lengths of all the segments with the red left endpoint and the blue right endpoint be equal to the sum of the lengths of all the segments with the blue left endpoint and the red right endpoint? (The endpoints of the segments in question are not necessarily adjacent points.)

Alexandr Gribalko

5 — 4. Consider an acute non-isosceles triangle. In a single step it is allowed to cut any one of available triangles into two triangles along its median. Is it possible that after a finite number of cuttings all triangles will be isosceles?

Egor Bakaev

5. A $2N \times 2N$ board is covered by non-overlapping domino pieces of 1×2 size. A lame rook (which can only move to a cell neighboring by side per 1 move) has visited each cell once on its route across the board. Let's call a move *longitudinal* if it is a transition from one cell of a domino to another cell of the same domino. What is:

1 — a) the maximum

4 — b) the minimum

possible number of longitudinal moves?

Boris Frenkin