44th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior A-Level Paper, Fall 2022

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

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points problems

4 — 1. One hundred friends, including Alice and Bob, live in several cities. Alice has found out the distance from her city to the city of each of the other 99 friends and added up these 99 numbers. Bob did the same thing as Alice. Alice got 1,000 km. What is the largest number that Bob could get? (Consider the cities as points on the plane; if two people live in the same city, the distance between their cities is considered zero).

Boris Frenkin

2. For the numbers 1, 19, 199, 1999, etc., they made individual cards, one card for each number, and put the corresponding number on each card.

2 — a) Is it possible to pick at least three cards so that the sum of the numbers on them equals a number in which all digits, except for a single digit, are twos?

3 — b) Suppose you have picked several cards so that the sum of the numbers on them equals the number, all the digits of which are twos, except for a single digit. What can this digit be?

Sergey Markelov

6 — 3. Baron Munchausen claims that he has drawn a polygon and a point inside it in such a way that any line passing through this point divides the polygon into three polygons. Could the baron be right in stating that?

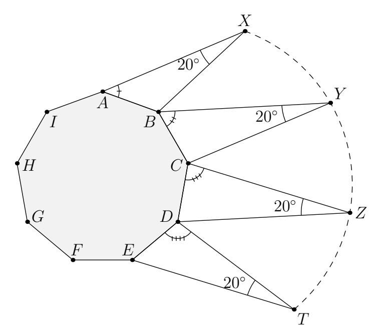
Tatiana Kazitsina

7 — 4. Let $n > 1$ be an integer. A rook stands in one of the cells of the infinite white checkered board. With each move the rook shifts across the board exactly for n cells, all vertically or all horizontally, painting the passed n cells black. After several moves in that manner, without visiting any cell twice, the rook returns to the starting cell. The black cells form a closed contour. Prove that the number of white cells inside this contour gives a remainder of 1 when divided by n.

Alexandr Gribalko

9 — 5. On the sides of a regular nonagon $ABCDEFGHI$ the triangles XAB, YBC, ZCD and TDE are constructed outside the nonagon. Given that the angles X, Y, Z, T of these triangles are 20° each. Among the angles XAB, YBC, ZCD, and TDE each next angle is 20° greater than the one listed before it. Prove that the points X, Y, Z, T lie on the same circle.

Egor Bakaev



10 — 6. Peter added a positive integer M to a positive integer N and noticed that the sum of the digits of the resulting integer is the same as the sum of the digits of N. Then he added M to the result again, and so on, and so on. Will he necessarily get again a number with the same sum of digits as the number N has?

Alexandr Shapovalov

7. It is known that among several banknotes of pairwise distinct face values (which are positive integers) there are exactly N of false ones. In a single check, the detector determines the sum of the face values of all real banknotes in an arbitrary set selected by us. Prove that in N checks all false banknotes can be detected, if

3 — a) $N = 2$;

8 — b) $N = 3$.

Sergey Tokarev

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Senior A-Level Paper, Fall 2022

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points	problems
5	1. What is the largest possible rational root of the equation $ax^2 + bx + c = 0$, where a, b, and c are positive integers that do not exceed 100? *Mikhail Evdokimov*
5	2. Consider two coprime integers p and q which are greater than 1 and differ from each other by more than 1. Prove that there exists a positive integer n such that $\operatorname{lcm}(p+n, q+n) < \operatorname{lcm}(p, q)$. *Mikhail Malkin*
6	3. Consider two concentric circles Ω and ω. The chord AD of the circle Ω is tangent to ω. Inside the minor segment AD of the disc with the boundary Ω, an arbitrary point P is selected. The tangent lines drawn from the point P to the circle ω intersect the major arc AD of the circle Ω at points B and C. The segments BD and AC intersect at the point Q. Prove that the segment PQ divides the segment AD into two equal parts. *Ivan Kukharchuk*
7	4. In a checkered square, there is a closed door between any two cells adjacent by side. The beetle starts from some cell and travels through cells, passing through doors. He opens a closed door in the direction he is moving and leaves that door open. Through an open door, the beetle can only pass in the direction the door is opened. Prove that if at any moment the beetle wants to return to the starting cell, he is able to do that. *Alexandr Perepechko*
8	5. In an infinite arithmetic progression of positive integers there are two integers with the same sum of digits. Will there necessarily be one more integer in the progression with the same sum of digits? *Alexandr Shapovalov*
	6. It is known that among several banknotes of pairwise distinct face values (which are positive integers) there are exactly N of false ones. In a single check, the detector determines the sum of the face values of all real banknotes in an arbitrary set selected by us. Prove that in N checks all false banknotes can be detected, if
2	a) $N = 2$;
7	b) $N = 3$. *Sergey Tokarev*
	7. There are N friends and a round pizza. It is allowed to make no more than 100 straight cuts without shifting the slices until all cuts are done, then distribute all the resulting slices among all the friends so that each of them gets a share of pizza having the same summary area. Is there a cutting which gives the above result when
5	a) $N = 201$;
5	b) $N = 400$? *Andrey Arzhantsev*