43rd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Spring 2022

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores.)

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points	problems
3	1. Two friends walked towards each other along a straight road. Each had a constant speed but one was faster than the other. At one moment each friend released his dog to run freely forward, the speed of each dog is the same and constant. Each dog reached the other person and then returned to its owner. Which dog returned to its owner the first, of the person who walks fast or who walks slow? *Alexandr Rubin*
4	2. Peter picked an arbitrary positive integer, multiplied it by 5, multiplied the result by 5, then multiplied the result by 5 again and so on. Is it true that from some moment all the numbers that Peter obtains contain 5 in their decimal representation? *Sergey Dorichenko*
5	3. The Fox and Pinocchio have grown a tree on the Field of Miracles with 11 golden coins. It is known that exactly 4 of them are counterfeit. All the real coins weigh the same, the counterfeit coins also weigh the same but are lighter. The Fox and Pinocchio have collected the coins and wish to divide them. The Fox is going to give 4 coins to Pinocchio, but Pinocchio wants to check whether they all are real. Can he check this using two weighings on a balance scale with no weights? *Alexandr Gribalko*
5	4. Consider a square $ABCD$. A point P was selected on its diagonal AC. Let H be the orthocenter of the triangle APD, let M be the midpoint of AD and N be the midpoint of CD. Prove that PN is orthogonal to MH. *Ivam Kukharchuk*
6	5. Let us call a 1×3 rectangle a tromino. Alice and Bob go to different rooms, and each divides a 20×21 board into trominos. Then they compare the results, compute how many trominos are the same in both splittings, and Alice pays Bob that number of dollars. What is the maximal amount Bob may guarantee to himself no matter how Alice plays? *Alexey Glebov*

43rd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior O-Level Paper, Spring 2022

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores.)

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points	problems
4	1. Peter picked a positive integer, multiplied it by 5, multiplied the result by 5, then multiplied the result by 5 again and so on. Altogether k multiplications were made. It so happened that the decimal representations of the original number and of all k resulting numbers in this sequence do not contain digit 7. Prove that there exists a positive integer such that it can be multiplied k times by 2 so that no number in this sequence contains digit 7. *Alexandr Gribalko*
4	2. The Fox and Pinocchio have grown a tree on the Field of Miracles with 8 golden coins. It is known that exactly 3 of them are counterfeit. All the real coins weigh the same, the counterfeit coins also weigh the same but are lighter. The Fox and Pinocchio have collected the coins and wish to divide them. The Fox is going to give 3 coins to Pinocchio, but Pinocchio wants to check whether they all are real. Can he check this using two weighings on a balance scale with no weights? *Alexandr Gribalko*
5	3. Let n be a positive integer. Let us call a sequence $a_1, a_2, \dots, a_n$ *interesting* if for any $i = 1, 2, \dots, n$ either $a_i = i$ or $a_i = i + 1$. Let us call an interesting sequence *even* if the sum of its members is even, and *odd* otherwise. Alice has multiplied all numbers in each odd interesting sequence and has written the result in her notebook. Bob, in his notebook, has done the same for each even interesting sequence. In which notebook is the sum of the numbers greater and by how much? (The answer may depend on n.) *Alexey Glebov*
5	4. Let us call a 1×3 rectangle a tromino. Alice and Bob go to different rooms, and each divides a 20×21 board into trominos. Then they compare the results, compute how many trominos are the same in both splittings, and Alice pays Bob that number of dollars. What is the maximal amount Bob may guarantee to himself no matter how Alice plays? *Alexey Glebov*
6	5. A quadrilateral $ABCD$ is inscribed into a circle ω with center O. The circumcircle of the triangle AOC intersects the lines AB, BC, CD and DA the second time at the points M, N, K and L respectively. Prove that the lines MN, KL and the tangents to ω at the points A и C all touch the same circle. *Azamat Mardanov*