43rd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2021

(The result is computed from the three problems with the highest scores.)

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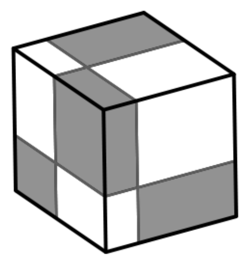
points problems

4 1. The Tournament of Towns is held once per year. This time the year of its autumn round is divisible by the number of the tournament: $2021 : 43 = 47$. How many times more will the humanity witness such a wonderful event?

Alexey Zaslavsky

5 2. A cube was split into 8 parallelepipeds by three planes parallel to its faces. The resulting parts were painted in a chessboard pattern. The volumes of the black parallelepipeds are 1, 6, 8, 12. Find the volumes of the white parallelepipeds.



5 3. A pirate has five purses with 30 coins in each purse contains only gold coins, another one cont the third one contains only bronze coins, and the remaining two ones contain 10 gold, 10 silver and 10 bronze coins each. It is allowed to simultaneously take one or several coins out of any purses (only once), and examine them. What is the minimal number of taken coins that is necessary to determine for sure the content of at least one purse?

Mikhail Evdokimov

5 4. A convex n-gon with $n > 4$ is such that if a diagonal cuts a triangle from it then this triangle is isosceles. Prove that there are at least 2 equal sides among any 4 sides of the n-gon.

Maxim Didin

5 5. There were 20 participants in a chess tournament. Each of them played with each other twice: once as white and once as black. Let us say that participant X is no weaker than participant Y if X has won at least the same number of games playing white as Y and also has won at least the same number of games playing black as Y. Do there exist for sure two participants A and B such that A is not weaker than B?

Boris Frenkin

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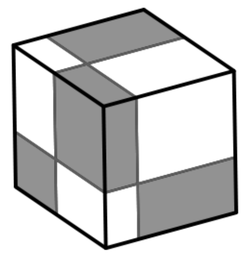
points	problems
3	1. Let us call a positive integer k *interesting* if the product of the first k primes is divisible by k. For example the product of the first two primes is $2 \cdot 3 = 6$, it is divisible by 2, hence 2 is an interesting integer. What is the maximal possible number of consecutive interesting integers? *Boris Frenkin*
4	2. A cube was split into 8 parallelepipeds by three planes parallel to its faces. The resulting parts were painted in a chessboard pattern. The volumes of the black parallelepipeds are 1, 6, 8, 12. Find the volumes of the white parallelepipeds.  *Oleg Smirnov*
6	3. In a checkered square of size 2021×2021 all cells initially are white. Ivan selects two cells and paints them black. At each step, all the cells that have at least one black neighbour by side are painted black simultaneously. Ivan selects the starting two cells so that the entire square is painted black as fast as possible. How many steps will this take? *Ivan Yashchenko*
6	4. Given a segment AB. Three points X, Y, Z are picked in the space so that ABX is an equilateral triangle and $ABYZ$ is a square. Prove that the orthocenters of all triangles XYZ obtained in this way belong to a fixed circle. *Alexandr Matveev*
6	5. Consider the segment $[0; 1]$. At each step we may split one of the available segments into two new segments and write the product of lengths of these two new segments onto a blackboard. Prove that the sum of the numbers on the blackboard never will exceed $1/2$. *Mikhail Lukin*