42nd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Junior O-Level Paper, Spring 2021
Grades 8 – 9 (ages 13-15)
(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

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points	problems
4	1. Is it possible that a product of 9 consecutive positive integers is equal to a sum of 9 consecutive (not necessarily the same) positive integers? *Boris Frenkin*
4	2. Let AX and BZ be altitudes of the triangle ABC. Let AY and BT be its angle bisectors. It is given that angles XAY and ZBT are equal. Does this necessarily imply that ABC is isosceles? *Jury*
4	3. Maria has a balance scale that can indicate which of its pans is heavier or whether they have equal weight. She also has 4 weights that look the same but have masses of 1001, 1002, 1004 and 1005 g. Can Maria determine the mass of each weight in 4 weighings? The weights for a new weighing may be picked when the result of the previous ones is known. *Jury*
	4.
3	a) Is it possible to split a square into 4 isosceles triangles such that no two are congruent?
3	b) Is it possible to split an equilateral triangle into 4 isosceles triangles such that no two are congruent? *Vladimir Rastorguev*
	5. There are several dominoes on a board such that each domino occupies two adjacent cells and none of the dominoes are adjacent by side or vertex. The bottom left and top right cells of the board are free. A token starts at the bottom left cell and can move to a cell adjacent by side: one step to the right or upwards at each turn. Is it always possible to move from the bottom left to the top right cell without passing through dominoes if the size of the board is
2	a) 100×101 cells;
4	b) 100×100 cells? *Nikolay Chernyatiev*

42nd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior O-Level Paper, Spring 2021

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

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points problems

1.

2 a) A convex pentagon is partitioned into three triangles by nonintersecting diagonals. Is it possible for centroids of these triangles to lie on a common straight line?

2 b) The same question for a non-convex pentagon.

Alexandr Gribalko

2.

2 a) Maria has a balance scale that can indicate which of its pans is heavier or whether they have equal weight. She also has 4 weights that look the same but have masses of 1000, 1002, 1004 and 1005 g. Can Maria determine the mass of each weight in 4 weighings? The weights for a new weighing may be picked when the result of the previous ones is known.

2 b) The same question when the left pan of the scale is lighter by 1 g than the right one, so the scale indicates equality when the mass on the left pan is heavier by 1 g than the mass on the right pan.

Alexey Tolpygo

5 3. For which n is it possible that a product of n consecutive positive integers is equal to a sum of n consecutive (not necessarily the same) positive integers?

Boris Frenkin

5 4. It is well-known that a quadratic equation has no more than 2 roots. Is it possible for the equation $\lfloor x^2 \rfloor + px + q = 0$ with $p \neq 0$ to have more than 100 roots? (By $\lfloor x^2 \rfloor$ we denote the largest integer not greater than x^2.)

Alexey Tolpygo

6 5. Let O be the circumcenter of an acute triangle ABC. Let M be the midpoint of AC. The straight line BO intersects the altitudes AA_1 and CC_1 at the points H_a and H_c respectively. The circumcircles of the triangles BH_aA and BH_cC have a second point of intersection K. Prove that K lies on the straight line BM.

Mikhail Evdokimov