42nd INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2020

(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

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points problems

3 1. Is it possible to select 100 points on a circle so that there are exactly 1000 right triangles with the vertices at selected points?

Sergey Dvoryaninov

2. A group of 8 players played several tennis tournaments between themselves using the single-elimination system, that is, the players are randomly split into pairs, the winners split into two pairs that play in semifinals, the winners of semifinals play in the final round. It so happened that after several tournaments each player had played with each other exactly once. Prove that

2 a) each player participated in semifinals more than once;

3 b) each player participated in at least one final.

Boris Frenkin

5 3. There are n stones in a heap. Two players play the game by alternatively taking either 1 stone from the heap or a prime number of stones which divides the current number of stones in the heap. The player who takes the last stone wins. For which n the first player has a strategy so that he wins no matter how the other player plays?

Fedor Ivlev

5 4. There is an equilateral triangle with side d and a point P such that the distances from P to the vertices of the triangle are positive numbers a, b, c. Prove that there exist a point Q and an equilateral triangle with side a, such that the distances from Q to the vertices of this triangle are b, c, d.

Alexandr Evnin

5 5. The director of a Zoo has bought eight elephants numbered by $1, 2, \ldots, 8$. He has forgotten their masses but he remembers that each elephant starting with the third one has the mass equal to the sum of the masses of two preceding ones. Suddenly the director hears a rumour that one of the elephants has lost his mass. How to perform two weighings on balance scales without weights to either find this elephant or make sure that this was just a rumour? (It is known that no elephant gained mass and no more than one elephant lost mass.)

Alexandr Gribalko

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Senior O-Level Paper, Fall 2020

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points problems

3 1. Each of the quadratic polynomials $P(x)$, $Q(x)$ and $P(x)+Q(x)$ with real coefficients has a repeated root. Is it guaranteed that those roots coincide?

Boris Frenkin

4 2. There were ten points $X_1, \ldots, X_{10}$ on a line in this particular order. Pete constructed an isosceles triangle on each segment X_1X_2, X_2X_3, $\ldots$, X_9X_{10} as a base with the angle α at its apex. It so happened that all the apexes of those triangles lie on a common semicircle with diameter X_1X_{10}. Find α.

Egor Bakaev

5 3. A positive integer number N is divisible by 2020. All its digits are different and if any two of them are swapped, the resulting number is not divisible by 2020. How many digits can such a number N have?

Sergey Tokarev

5 4. The sides of a triangle are divided by the angle bisectors into two segments each. Is it always possible to form two triangles from the obtained six segments?

Lev Emelyanov

5. There are 101 coins in a circle, each weights 10 g or 11 g. Prove that there exists a coin such that the total weight of the k coins to its left is equal to the total weight of the k coins to its right where

3 a) $k = 50$;

3 b) $k = 49$.

Alexandr Gribalko