41st INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Spring 2020

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores.)

---

points problems

4 — 1. The Quadrumland map is a 6×6 square where each square cell is either a kingdom or a disputed territory. There are 27 kingdoms and 9 disputed territories. Each disputed territory is claimed by those and only those kingdoms that are neighbouring with it (adjacent by an edge or a vertex). Is it possible that for each disputed territory the numbers of claims are different?

Mikhail Evdokimov

4 — 2. What is the maximum number of distinct integers in a row such that the sum of any 11 consequent integers is either 100 or 101?

Egor Bakaev

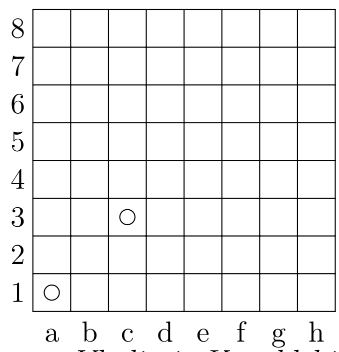
4 — 3. Let $ABCD$ be a rhombus, let $APQC$ be a parallelogram such that the point B lies inside it and the side AP is equal to the side of the rhombus. Prove that B is the orthocenter of the triangle DPQ.

Egor Bakaev

5 — 4. For some integer n the equation $x^2 + y^2 + z^2 - xy - yz - zx = n$ has an integer solution x, y, z. Prove that the equation $x^2 + y^2 - xy = n$ also has an integer solution x, y.

Alexandr Yuran

5 — 5. On the 8×8 chessboard there are two identical markers in the squares a1 and c3. Alice and Bob in turn make the following moves (the first move is Alice's): a player picks any marker and moves it horizontally to the right or vertically upwards through any number of squares. The aim of each player is to get to the square h8. Which player has a winning strategy no matter what does his opponent? (There may be only one marker on a square, the markers may not go through each other.)



Vladimir Kovaldzhi

41st INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Senior O-Level Paper, Spring 2020
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed up.)

---

points	problems
4	1. Is it possible to fill a 40×41 table with integers so that each integer equals the number of adjacent (by an edge) cells with the same integer? *Alexandr Gribalko*
4	2. Alice asserts that after her recent visit to Addis-Ababa she now has spent the New Year inside every possible hemisphere of Earth except one. What is the minimal number of places where Alice has spent the New Year? *Note:* we consider places of spending the New Year to be points on the sphere. A point on the border of a hemisphere does not lie inside the hemisphere. *Ilya Dumansky, Roman Krutovsky*
5	3. There are 41 letters on a circle, each letter is A or B. It is allowed to replace ABA by B and conversely, as well as to replace BAB by A and conversely. Is it necessarily true that it is possible to obtain a circle containing a single letter repeating these operations? *Maxim Didin*
	4. We say that a nonconstant polynomial $p(x)$ with real coefficients is *split into two squares* if it is represented as $a(x)+b(x)$ where $a(x)$ and $b(x)$ are squares of polynomials with real coefficients. Is there such a polynomial $p(x)$ that it may be split into two squares
2	a) in exactly one way;
3	b) in exactly two ways? *Note:* two splittings that differ only in the order of summands are considered to be the same. *Sergey Markelov*
5	5. Given are two circles which intersect at points P and Q. Consider an arbitrary line ℓ through Q, let the second points of intersection of this line with the circles be A and B respectively. Let C be the point of intersection of the tangents to the circles in those points. Let D be the intersection of the line AB and the bisector of the angle CPQ. Prove that all possible D for any choice of ℓ lie on a single circle. *Alexey Zaslavsky*