**41st INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS**
Junior O-Level Paper, Fall 2019
Grades 8 – 9 (ages 13-15)
*(The result is computed from the three problems with the highest scores.)*

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points problems

4 1. An illusionist performs the following trick: he lays a full deck of 52 cards in a row. On each step a spectator selects some integer number, and the card lying on the place with this number in the row is thrown away. However, each time after the number is selected the illusionist chooses from which side of the row it is counted. The illusionist announced beforehand that in the end only a definite card, namely the Three of Clubs, should remain. For which initial positions of the Three of Clubs the illusionist can guarantee the success of the trick?

*Alexey Voropaev*

4 2. Let $\omega$ be a circle with the center $O$ and $A$ and $C$ be two different points on $\omega$. For any third point $P$ of the circle let $X$ and $Y$ be the midpoints of the segments $AP$ and $CP$. Finally, let $H$ be the orthocenter (the point of intersection of the altitudes) of the triangle $OXY$. Prove that the position of the point $H$ does not depend on the choice of $P$.

*Artemiy Sokolov*

4 3. There is a row of 100 cells each containing a token. For 1 dollar it is allowed to interchange two neighbouring tokens. Also it is allowed to interchange with no charge any two tokens such that there are exactly 3 tokens between them. What is the minimum price for arranging all the tokens in the reverse order?

*Egor Bakaev*

5 4. There are given 1000 integers $a_1, \ldots, a_{1000}$. Their squares $a_1^2, \ldots, a_{1000}^2$ are written in a circle. It so happened that the sum of any 41 consecutive numbers on this circle is a multiple of $41^2$. Is it necessarily true that every integer $a_1, \ldots, a_{1000}$ is a multiple of 41?

*Boris Frenkin*

5 5. Basil has an unrestricted supply of straight bricks $1 \times 1 \times 3$ and $\Gamma$-shape bricks made of three cubes $1 \times 1 \times 1$. Basil filled a whole box $m \times n \times k$ with these bricks, where $m$, $n$ and $k$ are integers greater than 1. Prove that it was sufficient to use only $\Gamma$-shape bricks.

*Mikhail Evdokimov*

**41st INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS**

Senior O-Level Paper, Fall 2019

Grades 10 – 11 (ages 15 and older)

*(The result is computed from the three problems with the highest scores.)*

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| points | problems |
|---|---|
| 3 | 1. An illusionist performs the following trick: he lays a full deck of 52 cards in a row. On each step a spectator selects some integer number, and the card lying on the place with this number in the row is thrown away. However, each time after the number is selected the illusionist chooses from which side of the row it is counted. The illusionist announced beforehand that in the end only a definite card, namely the Three of Clubs, should remain. For which initial positions of the Three of Clubs the illusionist can guarantee the success of the trick? *Alexey Voropaev* |
| 4 | 2. Let $ABCDE$ be a convex pentagon such that $AE \parallel CD$ and $AB = BC$. Let $K$ be the intersection of the bisectors of the angles $A$ and $C$. Prove that $BK \parallel AE$. *Egor Bakaev* |
| 4 | 3. Any integer $x$ written on a blackboard may be replaced either with $3x + 1$ or with $[x/2]$ (the greatest integer not exceeding $x/2$). Prove that if the number 1 was written initially then an arbitrary positive integer can be obtained by some sequence of the above operations. *Vladislav Novikov* |
| 5 | 4. In a polygon, any two neighbouring sides are mutually perpendicular. We call any two of its vertices *disunited* if the bisectors of the angles at these vertices are mutually perpendicular. Prove that for each vertex the number of vertices disunited with it is even. *Mikhail Skopenkov* |
| 5 | 5. There is a row of 100 cells each containing a token. For 1 dollar it is allowed to interchange two neighbouring tokens. Also it is allowed to interchange with no charge any two tokens such that there are exactly 4 tokens between them. What is the minimum price for arranging all the tokens in the reverse order? *Egor Bakaev* |