40th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Junior O-Level Paper, Spring 2019
Grades 8 – 9 (ages 13-15)
(The result is computed from the three problems with the highest scores.)

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points	problems
3	1. Consider a sequence of positive integers with total sum 20 such that no number and no sum of a set of consecutive numbers is equal to 3. Is it possible for such a sequence to contain more than 10 numbers? *Alexandr Shapovalov*
4	2. Consider $2n+1$ coins lying in a circle. At the beginning, all the coins are heads up. Moving clockwise, $2n+1$ flips are performed: one coin is flipped, the next coin is skipped, the next coin is flipped, the next two coins are skipped, the next coin is flipped, the next three coins are skipped, the next coin is flipped and so on, until finally $2n$ coins are skipped and the next coin is flipped. Prove that at the end of this procedure, exactly one coin is heads down. *Vladimir Rastorguyev*
4	3. The product of two positive integers m and n is divisible by their sum. Prove that $m+n \leqslant n^2$. *Boris Frenkin*
5	4. Isosceles triangles with a fixed angle α at the vertex opposite to the base are being inscribed into a rectangle $ABCD$ so that this vertex lies on the side BC and the vertices of the base lie on the sides AB and CD. Prove that the midpoints of the bases of all such triangles coincide. *Igor Zhizhilkin*
5	5. A magician and his assistant are performing the following trick. There is a row of 12 empty closed boxes. The magician leaves the room, and a person from the audience hides a coin in each of two boxes of his choice, so that the assistant knows which boxes contain coins. The magician returns and the assistant is allowed to open one box that does not contain a coin. Next, the magician selects four boxes, which are then simultaneously opened. The goal of the magician is to open both boxes that contain coins. Devise a method that will allow the magician and his assistant to always successfully perform the trick. *Konstantin Knop*

40th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Senior O-Level Paper, Spring 2019
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores.)

points	problems
4	1. The distances from a certain point inside a regular hexagon to three of its consecutive vertices are equal to 1, 1 and 2, respectively. Determine the length of this hexagon's side. *Mikhail Evdokimov*
4	2. Consider two positive integers a and b such that $a^{n+1} + b^{n+1}$ is divisible by $a^n + b^n$ for infinitely many positive integers n. Is it necessarily true that $a = b$? *Boris Frenkin*
4	3. Prove that any triangle can be cut into 2019 quadrilaterals such that each quadrilateral is both inscribed and circumscribed. *Nairi Sedrakyan*
5	4. A magician and his assistant are performing the following trick. There is a row of 13 empty closed boxes. The magician leaves the room, and a person from the audience hides a coin in each of two boxes of his choice, so that the assistant knows which boxes contain coins. The magician returns and the assistant is allowed to open one box that does not contain a coin. Next, the magician selects four boxes, which are then simultaneously opened. The goal of the magician is to open both boxes that contain coins. Devise a method that will allow the magician and his assistant to always successfully perform the trick. *Igor Zhizhilkin*
5	5. Consider a sequence of positive integers with total sum 2019 such that no number and no sum of a set of consecutive numbers is equal to 40. What is the greatest possible length of such a sequence? *Alexandr Shapovalov*