40th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2018

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
4	**1.** A circle passing through vertex B and the midpoint of the hypotenuse AC of the right triangle ABC intersects two other sides at points M and N. If $AC = 2MN$ prove that the points M and N are the midpoints of the sides. *Mikhail Evdokimov*
4	**2.** Determine all positive integers n for which the set of numbers 1, 2, $\ldots$, $2n$ can be split into pairs so that the product of sums of every pair is a perfect square. *Folklore*
	3. A grid rectangle 7×14 is split along grid lines into 2×2 squares and 3-cell corners. Is it possible that:
1	a) the number of squares and corners is the same;
3	b) there are more squares than corners? *Mikhail Evdokimov*
5	**4.** Among 5 Kate's coins identical in appearance three are real and weigh the same, and two are fake: one of them weighs more than a real one, and another fake coin weighs less than a real one by the same quantity. Kate can ask an expert to perform three weighings of her choice, on a simple balance. Then the expert reports the results to Kate. Could Kate choose the weighings so that she would be able to determine both fake coins and state which of them is heavier? A simple balance shows which of two sides is heavier/ higher or if they are balanced. *Rustem Zhenodarov*
5	**5.** A nine-digit integer is called *beautiful* if all of its digits are different. Prove that there are at least 1000 beautiful multiples of 37. *Mikhail Evdokimov*

40th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS
Senior O-Level Paper, Fall 2018
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores.)

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points	problems
3	**1.** Is it possible to place a line segment inside a regular pentagon so that it would be seen under the same angle from each vertex of the pentagon? (From a point A a segment XY is seen under angle XAY.) *Egor Bakaev, Sergey Dvoryaninov*
4	**2.** Determine all positive integers n for which the set of numbers 1, 2, . . . , $2n$ can be split into pairs so that the product of sums of every pair is a perfect square. *Folklore*
5	**3.** The angle A of the parallelogram $ABCD$ is acute. A point N is chosen on the side AB so that $CN = AB$. If the circumscribed circle of the triangle CBN is tangent to the line AD, prove that D is the point of tangency. *Mikhail Evdokimov*
5	**4.** A nine-digit integer is called *beautiful* if all of its digits are different. Prove that there are at least 2018 beautiful multiples of 37. *Mikhail Evdokimov*
5	**5.** Pete is placing 500 kings on a 100×50 board so that none of them attacks one another. Basil is placing 500 kings on white cells of a 100×100 chessboard so that none of them attacks one another. Who has more ways to place the kings? *Egor Bakaev*