39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Spring 2018

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points problems

1. Six rooks are placed on a 6×6 board, so that no one is under attack. Each unoccupied square is coloured according to the following rule: if all the rooks that attack this square are the same distance away from it then this square is coloured red, and blue otherwise. Is it possible that all the unoccupied squares are coloured

1 a) red;

2 b) blue?

Igor Akulich

4 2. Let K be a point on the hypotenuse AB of a right triangle ABC, and L be a point on the side AC, such that $AK = AC$ and $BK = LC$. Let M be the intersection point of the segments BL and CK. Prove that the triangle CLM is isosceles.

Egor Bakaev

3. In each cell of a 4×4 square there is an integer number. The sum of the numbers in each column and each row is the same. Seven of the numbers are known, while the rest are hidden (see figure). Is it possible to uniquely determine

1	?	?	2
?	4	5	?
?	6	7	?
3	?	?	?

2 a) at least one of the hidden numbers;

2 b) at least two of the hidden numbers?

Egor Bakaev

4 4. Three natural numbers are such that each of them is divisible by the greatest common divisor of the other two numbers, and the least common multiple of any two is divisible by the third number. Are these three numbers necessarily equal?

Boris Frenkin

5 5. Thirty points are marked on the plane such that no three lie on the same line. Seven red lines are drawn so that they do not contain any of the marked points. Is it possible that any segment connecting two marked points intersects at least one red line?

Pavel Kozhevnikov

39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Senior O-Level Paper, Spring 2018
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores.)

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points	problems
3	1. The bisector and altitude drawn from one vertex of a triangle divide the opposite side into three segments. Can it occur that it is possible to construct a triangle out of those three segments? *Mikhail Evdokimov*
4	2. Four natural numbers are such that each of them is divisible by the greatest common divisor of the other three numbers, and the least common multiple of any three is divisible by the fourth number. Prove that the product of these four numbers is a perfect square. *Boris Frenkin*
4	3. Two circles with centers at O_1 and O_2 are outer tangent, touching at the point T. An outer common tangent is drawn, touching the first circle at point A and the second at point B. The common tangent passing through the point T intersects the line AB at point M. Let AC be a diameter of the first circle. Prove that the segments CM and AO_2 are orthogonal. *Pavel Kozhevnikov*
5	4. In the corner of an 8×8 chessboard there is a chip. Peter and Basil take turns moving the chip. Pete starts first, and on his turn he makes a move as a chess queen (only the last square is considered passed). Basil on his turn makes a double move as a chess king (both squares are considered passed). The initial square is also considered passed. The chip cannot be moved to a square already passed. The player who cannot make a move loses. Which of the boys can play so that he will always win, no matter how his opponent will move? *Alexandr Shapovalov*
5	5. At each vertex of a polyhedron, exactly three faces meet. Each face of this polyhedron is coloured red, yellow or blue. The vertices, where the faces of all three colours meet, are called multicoloured. Prove that the number of multicoloured vertices is even. *Egor Bakaev, Alexandr Gribalko, Inessa Raskina*