

39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior A-Level Paper, Spring 2018

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores.)

points problems

- 4 1. Thirty nine nonzero numbers are written in a row. The sum of any two neighbouring numbers is positive, while the sum of all the numbers is negative. Is the product of all these numbers negative or positive?

Boris Frenkin

- 5 2. Aladdin has several gold ingots, and sometimes he asks the Genie to give him more. The Genie first adds a thousand ingots, but then takes half of the total number. Could it be possible that after asking the Genie for gold ten times, the number of Aladdin's gold ingots increased, assuming that each time the Genie took half, he took an integer number of ingots?

Alexandr Perepechko

- 6 3. Do there exist 2018 positive irreducible fractions, each with a different denominator, so that that the denominator of the difference of any two (after reducing the fraction) is less than the denominator of any of the initial 2018 fractions?

Maxim Didin

- 6 4. Let O be the center of the circumscribed circle of the triangle ABC . Let AH be the altitude in this triangle, and let P be the base of the perpendicular drawn from point A to the line CO . Prove that the line HP passes through the midpoint of the side AB .

Egor Bakaev

- 8 5. There are 100 houses in the street, divided into 50 pairs. In each pair houses are right across the street one from another. On the right side of the street the houses have even numbers, while the houses on the left side have odd numbers. On both sides of the street the numbers increase from the beginning to the end of the street, but are not necessarily consecutive (some numbers may be omitted). For each house on the right side of the street, the difference between its number and the number of the opposite house was computed, and it turned out that all these values were different. Let n be the greatest number of a house on this street. Find the smallest possible value of n .

Maxim Didin

- 10 6. In the land of knights (who always tell the truth) and liars (who always lie), 10 people sit at a round table, each at a vertex of an inscribed regular 10-gon, at least one of them is a liar. A traveler can stand at any point outside the table and ask the people: "What is the distance from me to the nearest liar at the table?" After that each person at the table gives him an answer. What is the minimal number of questions the traveler has to ask to determine which people at the table are liars? (Both the people at the table and the traveler should be considered as points, and everyone can compute the distance between any two points)

Maxim Didin

- 12 7. You are in a strange land and you don't know the language. You know that "!" and "?" stand for addition and subtraction, but you don't know which is which. Each of these two symbols can be written between two arguments, but for subtraction you don't know if the left argument is subtracted from the right or vice versa. So, for instance, $a?b$ could mean any of $a - b$, $b - a$, and $a + b$. You don't know how to write any numbers, but variables and parenthesis work as usual. Given two arguments a and b , how can you write an expression that equals $20a - 18b$?

Nikolay Belukhov

39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Senior A-Level Paper, Spring 2018

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

points problems

- 4 1. Aladdin has several gold ingots, and sometimes he asks the Genie to give him more. The Genie first adds a thousand ingots, but then takes half of the total number. Could it be possible that after asking the Genie for gold ten times, the number of Aladdin's gold ingots increased, assuming that each time the Genie took half, he took an integer number of ingots?
Alexandr Perepechko
- 5 2. Do there exist 2018 positive irreducible fractions, each with a different denominator, so that that the denominator of the difference of any two (after reducing the fraction) is less than the denominator of any of the initial 2018 fractions?
Maxim Didin
- 6 3. One hundred different numbers are written in the cells of a 10×10 square. Each step you can select a rectangle formed by the cells, and for every cell inside swap its number with the number in the cell opposite to it with respect to the center of the rectangle ("turn the rectangle by 180° ") Is it always possible to arrange the numbers in the square so that they increase in every row from left to right, and in every column from bottom to top, in no more than 99 turns?
Alexandr Shapovalov
- 4 4. An equilateral triangle lying in the plane α is orthogonally projected onto a plane β , which is not parallel to α . The resulting triangle is again orthogonally projected onto a plane γ , and it turned out that the image is again an equilateral triangle. Prove that
- 4 a) the angle between the planes α and β is equal to the angle between the planes β and γ ;
4 b) the plane β intersects the plane α and γ along perpendicular lines.
Lev Emelyanov
- 10 5. You are in a strange land and you don't know the language. You know that "!" and "?" stand for addition and subtraction, but you don't know which is which. Each of these two symbols can be written between two arguments, but for subtraction you don't know if the left argument is subtracted from the right or vice versa. So, for instance, $a?b$ could mean any of $a - b$, $b - a$, and $a + b$. You don't know how to write any numbers, but variables and parenthesis work as usual. Given two arguments a and b , how can you write an expression that equals $20a - 18b$?
Nikolay Belukhov
- 10 6. The quadrilateral $ABCD$ is inscribed into a circle S . Let P be the intersection point of the rays BA and CD . Let U and V be the intersection points of the line, which passes through P and is parallel to the tangent to S at the point D , with the tangents to S at points A and B respectively. Prove that the circumcircle of the triangle CUV is tangent to the circle S .
Alexey Zaslavsky
- 12 7. The King decided to reward a group of n wizards. They will be placed in line one after another (so that they will look in the same direction), and each will have a black or white hat. Every wizard will see the hats of the people in front of him. Starting from the back of the line, each wizard in turn will name a colour (black or white) and a natural number on his choice. At the end the King counts the number of wizards who named the colour identical with the colour of their hat, and rewards all the wizards with the same number of days with bonus pay. The wizards are allowed to decide on their collective strategy prior to forming the line, but they know that just k of them are insane (it is not known who is insane). An insane wizard names a white or black colour and a natural number regardless of the collective strategy. What is the maximal number of days with bonus pay which can be guaranteed by the collective strategy, independently of where the insane wizards will be placed in the line?
Ivan Mitrofanov