39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2017

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores.)

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points	problems
3	**1.** There are five nonzero numbers. For each pair of the numbers, their sum and product have been computed. It turned out that five of the sums are positive and the other five are negative. Find the number of positive and negative products. *Boris Frenkin*
4	**2.** Do there exist 99 consecutive natural numbers such that the smallest one is divisible by 100, the next by 99, the third by 98, ..., and the last one by 2? *Pavel Kozhevnikov*
4	**3.** One hundred coins, identical in appearance, lie in a row. Among them are exactly 26 fake coins, which lie consecutively. The weights of all true coins are the same while each fake coin is lighter that a true one (fake coins do not have necessarily equal weights). Find at least one fake coin with a single use of a two-cup scale. *Rustem Zhenodarov*
5	**4.** A treasure is buried in one of the cells of an 8×8 square. You start with a metal detector in one of the corner cells, and can move to cells, which are adjacent by side. The metal detector beeps if you are standing on the cell with the treasure or on a cell adjacent to it by side. Is it possible to identify the cell with the treasure by making no more than 26 moves? *Mikhail Evdokimov*
5	**5.** A circle of radius 1 is drawn on a chessboard so that it contains an entire white cell (the sides of the cells are equal to 1). Prove that the parts of the circumference passing through white cells have total length at most $1/3$ of the total length of the circumference. *Mikhail Evdokimov*

39th INTERNATIONAL MATHEMATICAL TOURNAMENT OF TOWNS
Senior O-Level Paper, Fall 2017
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores.)

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points problems

4 **1.** Do there exist non-integer numbers x and y such that $\{x\} \cdot \{y\} = \{x + y\}$? (Here $\{x\}$ is the fractional part of x.)

Mikhail Evdokimov

4 **2.** Let CL be a bisector of triangle ABC. The perpendicular at the midpoint to side AC intersects the interval CL at point K. Prove that the circumscribed circles of the triangles ABC and AKL are tangent.

Mikhail Panov

4 **3.** There are 21 non-zero numbers. For each pair of the numbers, their sum and product have been computed. It turned out that half of all the sums are positive and the other half are negative. What is the maximum possible number of positive products?

Boris Frenkin, Sergey Kudrya

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2 a) Is it possible for some sphere to intersect the faces of a regular tetrahedron along circles of radius 1, 2, 3 and 4?

3 b) Same question for a sphere of radius 5.

Mikhail Evdokimov

5 **5.** In the bottom left corner of a 100×100 chessboard there is a checker. Alternating horizontal and vertical moves (every move is to an adjacent square; the first move is horizontal), it first reached the top left corner, and then the top right corner. Prove that there are two adjacent (by side) squares a and b such that the checker moved from a to b at least twice.

Alexandr Gribalko