38th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior O-Level Paper, Spring 2017

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
3	1. Find the smallest natural number that is divisible by 2017 and whose decimal notation begins with 2016. *Mikhail Evdokimov*
4	2. Prove that the graph of any quadratic trinomial with one root and leading coefficient 1 contains a point (p, q) such that the trinomial $x^2 + px + q$ also has one root. *Boris Frenkin*
5	3. An acute-angled triangle ABC is such that $\angle A = 60°$. A billiard ball goes from vertex A along the bisector of angle A, reflects about the side BC according to the law "the angle of reflection equals the angle of incidence" and continues along a line without any further reflections. Prove that the path of the ball contains the circumcenter of triangle ABC. *Alexandr Kuznetsov*
5	4. A hundred children of distinct height stand in a line. At each step, a group of 50 consecutive children are chosen and rearranged in an arbitrary way. Show that in 6 such steps, the children can be arranged so that their heights decrease from left to right (regardless of the initial arrangement). *Ilya Bogdanov*
	5.
2	a) Given a 10-gon (not necessary convex), draw circles with its sides as diameters. Is it possible that all these circles pass through a point which is not a vertex of this 10-gon?
3	b) Solve the same problem for a 11-gon. *Egor Bakaev*

38th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS
Senior O-Level Paper, Spring 2017
Grades 10 – 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores.)

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points	problems
3	1. Given a regular 12-gon $A_1A_2\dots A_{12}$, is it possible to choose 7 vectors among $\overrightarrow{A_1A_2}$, $\overrightarrow{A_2A_3}$, $\dots$, $\overrightarrow{A_{11}A_{12}}$, $\overrightarrow{A_{12}A_1}$ such that their sum is the zero vector? *Mikhail Murashkin*
4	2. Given two concentric circumferences and a point A inside the inner circumference. The angle of a size α with vertex A cuts an arc on each circumference. Prove that if the arc of the outer circumference has the angular size α then the arc of the inner circumference also has the angular size α. *Egor Bakaev*
5	3. Each cell of a square 1000×1000 table contains a number. It is known that the sum of the numbers in each rectangle of area S with sides along the borders of cells, contained in the table, is the same. Find all values of S which guarantee that all the numbers in the table are equal. *Egor Bakaev*
5	4. Ten children of distinct height stand in a circle. Sometimes, one of them moves to a new place in the circle between two children. The children want to be arranged as soon as possible by increasing height clockwise (from the lowest to the highest child). What is the minimal number of moves sufficient for this regardless of initial arrangement of children? *Egor Bakaev*
6	5. The graphs of two quadratic trinomials intersect in two points. In each of these points, the tangents to the graphs are perpendicular. Is it necessarily true that the graphs have a common axis of symmetry? *Alexey Zaslavsky*