INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior A-Level Paper, Spring 2017.

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points problems

5 1. A chess tournament had 10 participants. Each round, the participants split into pairs, and each pair played a game. In total, each participant played with every other participant exactly once, and in at least half of the games both the players were from the same town. Prove that during each round there was a game played by two participants from the same town.

Boris Frenkin

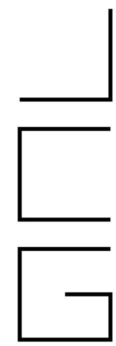
2.

1 a) Is it possible to draw a polygon on a grid paper and divide it into two equal parts by a cut of the shape as shown on the upper drawing?

2 b) Solve the same problem for the cut shown on the middle drawing.

4 c) Solve the same problem for the cut shown on the bottom drawing.

(In all of these problems the cut is inside the polygon, only the ends of the cut lie on the boundary. The sides of the polygons and the cuts must lie on the sides of the cells. The small links of the cuts are twice as short as the large ones.)



Yuri Markelov, 7th grade student

3. From given positive numbers, the following infinite sequence is defined: a_1 is the sum of all original numbers, a_2 is the sum of the squares of all original numbers, a_3 is the sum of the cubes of all original numbers, and so on (a_k is the sum of the k-th powers of all original numbers).

4 a) Can it happen that $a_1 > a_2 > a_3 > a_4 > a_5$ and $a_5 < a_6 < a_7 < \dots$?

4 b) Can it happen that $a_1 < a_2 < a_3 < a_4 < a_5$ and $a_5 > a_6 > a_7 > \dots$?

Alexey Tolpygo

8 4. All the sides of the convex hexagon $ABCDEF$ are equal. In addition, $AD = BE = CF$. Prove that a circle can be inscribed into this hexagon.

Boyan Obukhov

8 5. There is a set of control weights, each of them weighs a non-integer number of grams. Any integer weight from 1 g to 40 g can be balanced by some of these weights (the control weights are on one balance pan, and the measured weight on the other pan).What is the least possible number of the control weights?

Alexandr Shapovalov

10 6. A grasshopper can jump along a checkered strip for 8, 9 or 10 cells in any direction. A natural number n is called jumpable if the grasshopper can start from some cell of a strip of length n and visit every cell exactly once. Find at least one non-jumpable number $n > 50$.

Egor Bakaev

7. 1×2 dominoes are placed on an 8×8 chessboard without overlapping. They may partially stick out from the chessboard but the center of each domino must be strictly inside the chessboard (not on its border). Place on the chessboard in such a way:

6 a) at least 40 dominoes,

3 b) at least 41 dominoes,

3 c) more than 41 dominoes.

Mikhail Evdokimov

INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Senior A-Level Paper, Spring 2017.

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
4	1. On the plane, there is a triangle and ten lines. Every line is equidistant from two of the triangle's vertices. Prove that either at least two of these lines are parallel or at least three of them pass through a common point. *Sergey Markelov*
	2. From given positive numbers, the following infinite sequence is defined: a_1 is the sum of all original numbers, a_2 is the sum of the squares of all original numbers, a_3 is the sum of the cubes of all original numbers, and so on (a_k is the sum of the k-th powers of all original numbers).
3	a) Can it happen that $a_1 > a_2 > a_3 > a_4 > a_5$ and $a_5 < a_6 < a_7 < \ldots$?
3	b) Can it happen that $a_1 < a_2 < a_3 < a_4 < a_5$ and $a_5 > a_6 > a_7 > \ldots$? *Alexey Tolpygo*
7	3. Basil claims that he dissected a convex polyhedron that has only triangular and hexagonal faces into two parts, which he rearranged into a cube. Can Basil's claim be true? *Mikhail Evdokimov*
8	4. Pete colored each cell of a 1000×1000 grid square by one of ten colors. He also created a grid polygon F consisting of 10 cells such that no matter how you place it on the square along grid lines, it would cover all ten different colors. Is it necessary for F to be a rectangle? *Egor Bakaev*
9	5. In a triangle ABC with $\angle A = 45°$, AM is a median. Line b is symmetrical to line AM with respect to altitude BB_1, and line c is symmetrical to line AM with respect to altitude CC_1. Lines b and c meet at point X. Prove that $AX = BC$.
10	6. Find all positive integers n such that for any integer $k \geq n$ there is a number divisible by n and with the sum of digits equal to k. *Alexandr Kuznetsov, Ivan Losev*
12	7. Each of 36 gangsters belongs to several gangs. There are no two gangs with the same roster. Gangsters from the same gang are all allies. If a gangster does not belong to a gang, he has at least one enemy in this gang. What is the greatest possible number of gangs? *Folklore, suggested by Lev Shabanov*