38th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2016

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points problems

4 1. Do there exist 5 positive integers such that all their pairwise sums end with different digits?

Mikhail Evdokimov

4 2. Four points are marked on a line, and one point is marked outside the line. Among 6 triangles with vertices at these points, what is the greatest possible number of isosceles triangles?

Egor Bakaev

3. On a circle, 100 points are marked and numbered from 1 to 100 in some order.

2 a) Prove that these points can be paired so that the segments joining the points in pairs do not intersect, and that sums in pairs are odd.

2 b) Is it always possible to pair these points so that the segments joining the points in pairs do not intersect, and that sums in pairs are even?

Pavel Kozhevnikov

5 4. Suppose $ABCD$ is a parallelogram. Let K be a point such that $AK = BD$ and point M be the midpoint of CK. Prove that $\angle BMD = 90°$.

Egor Bakaev

5 5. A hundred bear-cubs picked up berries in a forest. The youngest bear-cub got one berry, the second youngest got 2 berries, the third youngest got 4 berries, and so on; the eldest cub got 2^{99} berries. They meet a fox who suggests to divide the berries 'fairly'. The fox chooses two bear-cubs and divides their berries equally between them, but if one berry is left over then the fox eats it. The fox proceeds in such a way until all bear-cubs have the same number of berries. What is the least possible number of berries that fox can leave for cubs?

Egor Bakaev

38th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Senior O-Level Paper, Fall 2016

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
4	1. Two parabolas with different vertices are the graphs of quadratic trinomials with leading coefficients p and q. The vertex of each parabola lies on the other parabola. What are the possible values of $p + q$? *Nairi Sedrakyan*
5	2. Hundred points are marked on a line, and one more is marked outside the line. Among the triangles with vertices at these points, what is the greatest possible number of isosceles triangles? *Egor Bakaev*
5	3. A hundred bear-cubs picked up berries in a forest. The youngest bear-cub got one berry, the second youngest got 2 berries, the third youngest got 4 berries, and so on; the eldest cub got 2^{99} berries. They meet a fox who suggests to divide the berries 'fairly'. The fox chooses two bear-cubs and divides their berries equally between them, but if one berry is left over then the fox eats it. The fox proceeds in such a way until all bear-cubs have the same number of berries. What is the greatest possible number of berries that the fox can eat? *Egor Bakaev*
5	4. Pete has drawn a polygon consisting of 100 cells on a grid paper. This polygon can be dissected along grid lines both into 2 congruent polygons and into 25 congruent polygons. Is it always true that this polygon can be also dissected along grid lines into 50 congruent polygons? *Egor Bakaev*
6	5. Prove that in a right-angled triangle, the orthocenter of the triangle formed by the points of tangency of the incircle with the sides lies on the altitude drawn to the hypotenuse. *Alexey Zaslavsky*