INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior A-Level Paper, Fall 2016.

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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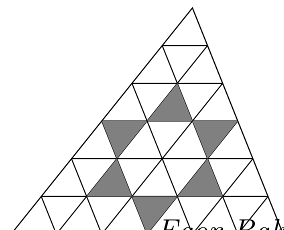
points	problems
5	1. Each of ten boys has 100 biscuits in his plate. Instead of eating, they play a game. Each step, one of the boys gives one biscuit from his plate to each of the other boys. What is the minimal number of steps needed to make the number of biscuits different for all the boys? *Nikolay Cherniatyev*
5	2. Each cell of an 8×8 board contains a positive integer. For all dissections of the board into tiles consisting of two cells adjacent by a side, the sums of integers in all tiles are different. Is it possible that the greatest integer on the board does not exceed 32? *Nikolay Cherniatyev*
6	3. An arbitrary triangle is dissected into congruent triangles by lines parallel to its sides (as is shown in the picture). Prove that the orthocenters of six painted triangles are concyclic.  *Egor Bakaev*
8	4. A square box of chocolates is divided into 49 equal square cells, each containing either dark or white chocolate. At each move Alex eats two chocolates of the same kind if they are in adjacent cells (by side or vertex). What is the maximal number of chocolates Alex can eat regardless of distribution of chocolates in the box? *Alexandr Kuznetsov*
8	5. Suppose three red and three blue cards contain distinct positive numbers. The cards of some colour contain the pairwise sums of some three numbers, and the cards of another colour contain the pairwise products of the same three numbers. Is it always possible to determine these three numbers? *Boris Frenkin*
9	6. Let $A_1A_2\ldots A_{2n}$ with $n \geq 5$ be a regular $2n$-gon with the center at O. Diagonals A_2A_{n-1} and A_3A_n intersect at F, diagonals A_1A_3 and A_2A_{2n-2} intersect at P. Prove that $PF = PO$. *Maxim Timokhin*
	7.
5	a) A questionnaire contains 20 questions which admit two possible answers. It is known that for any collection of 10 questions and any combination of answers to these questions, some person has given precisely these answers to these questions. Does this imply that there exist two people which gave different answers to all the questions?
6	b) The same problem for 12 possible answers to each question. *Ivan Mitrofanov, Alexey Kanel-Belov*

INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Senior A-Level Paper, Fall 2016.

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
5	1. Each of 100 boys has 100 biscuits in his plate. Instead of eating, they play a game. At each step, one of the boys chooses a group of boys (including at least one boy) and gives one biscuit to each of them. What is the minimal number of steps needed to make the number of biscuits different for all the boys? *The Jury of the Tournament of Towns, after Nikolay Cherniatyev*
5	2. Each cell of an 8×8 board contains a positive integer. For all dissections of the board into tiles consisting of two cells adjacent by a side, the sums of integers in all tiles are different. Is it possible that the greatest integer on the board does not exceed 32? *Nikolay Cherniatyev*
7	3. A quadrilateral $ABCD$ is inscribed into a circle centered at point O not belonging to the diagonals of the quadrilateral. The circle passing through points A, O and C contains the midpoint of BD. Prove that the circle passing through points B, O and D contains the midpoint of AC. *Alexey Zaslavsky*
8	4. Suppose 2016 red and 2016 blue cards contain distinct positive numbers. The cards of one colour contain the pairwise sums of some 64 numbers, and the cards of the other colour contain the pairwise products of the same 64 numbers. Is it always possible to determine the colour of cards that contain the pairwise sums? *Boris Frenkin*
9	5. Is it possible to cut the 1×1 square into two parts which can cover a disk of diameter greater than 1? *Alexandr Shapovalov*
9	6. Alice and Bob play the following game: Alice chooses a polynomial $P(x)$ with integer coefficients. At each his move, Bob pays 1 dollar to Alice and reports her some integer a. He cannot choose the same integer a twice. Alice responds by returning the number of integer solutions of the equation $P(x) = a$. Bob wins when Alice repeats the number already reported by her (not necessarily at the preceding move). Determine the minimal amount of cash sufficient for Bob to win the game regardless of the polynomial chosen by Alice. *Anant Mungal (India)*
12	7. Finite number of frogs occupy distinct integer points on the real line. At each move, a single frog jumps by 1 to the right so that all frogs again occupy distinct points. For some initial configuration, the frogs can make n moves in m ways. Prove that if they jump by 1 to the left (instead of right) then the number of ways to make n moves is also m. *Fedor Petrov*