37th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior O-Level Paper, Spring 2016

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
3	1. Twenty children stand in a circle (both boys and girls are present). For each boy, his clockwise neighbour is in a blue T-shirt, and for each girl, her counterclockwise neighbour is in a red T-shirt. Is it possible to determine the precise number of boys in the circle? *Egor Bakaev*
4	2. Given an acute-angled triangle ABC wth $\angle C = 60^\circ$. Let H be the point of intersection of its altitudes. The circle of radius HC centered at H meets the lines CA and CB for the second time at points M and N respectively. Prove that lines AN and BM are parallel or coincide. *Alexandr Zimin*
5	3. Is it possible that the sum and the product of 2016 integers are both equal to 2016? *Folklore, suggested by Mikhail Evdokimov*
5	4. On a checkered square 10×10 the cells of the upper left 5×5 square are black and all the other cells are white. What is the maximal n such that the original square can be dissected (along the borders of the cells) into n polygons such that in each of them the number of black cells is three times less than the number of white cells? (The polygons need not be congruent or even equal in area.) *Egor Bakaev*
5	5. On a list of paper, a blue triangle is drawn. A median, a bisector and an altitude of this triangle (not necessarily from three distinct vertices) are drawn red. The triangle dissects into several parts. Is it possible that one of these parts is a regular triangle with red sides? *Mikhail Evdokimov*

37th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS
Senior O-Level Paper, Spring 2015
Grades 10 – 11 (ages 15 and older)

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points	problems
4	1. A point inside a convex quadrilateral is connected with all the vertices and with four more points such that each side contains one of them. The quadrilateral dissects into eight triangles with equal radii of circumcircles. Prove that the original quadrilateral is cyclic. *Egor Bakaev*
4	2. Is it possible that the sum and the product of 2016 integers are both equal to 2016? *Folklore, suggested by Mikhail Evdokimov*
4	3. On a checkered square 10×10 the cells of the upper left 5×5 square are black and all the other cells are white. What is the maximal n such that the original square can be dissected (along the borders of the cells) into n polygons such that in each of them the number of black cells is three times less than the number of white cells? (The polygons need not be congruent or even equal in area.) *Egor Bakaev*
6	4. A firm has fixed its expenses in roubles for 100 articles in its budget. Each of the 100 numbers obtained has at most two digits after the decimal point. Each bookkeeper chooses two numbers in his copy of this list, adds them, removes the digits of the sum after the decimal point (if any) and writes the result instead of those two numbers. The new list consisting of 99 numbers is treated in the same way and so on, until a single integer remains. The final results happened to be different for all bookkeepers. What is the maximal possible number of the bookkeepers? *Mikhail Evdokimov*
	5. At each of the 12 edges of a cube, the midpoint is marked. Does a sphere necessarily contain all these points if it contains
3	a) at least 6 of marked points;
3	b) at least 7 of marked points? *Mikhail Evdokimov*