37th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Junior O-Level Paper, Fall 2015

Grades 8 – 9 (ages 13-15)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
4	1. Is it true that every positive integer can be multiplied by one of integers 1, 2, 3, 4 or 5 so that the resulting number starts with 1? *Egor Bakaev*
4	2. A rectangle is split into equal non-isosceles right-angled triangles (without gaps or overlaps). Is it true that any such arrangement contains a rectangle made of two such triangles? *Egor Bakaev*
5	3. Three players play the game "rock-paper-scissors". In every round, each player simultaneously shows one of these shapes. Rock beats scissors, scissors beat paper, while paper beats rock. If in a round exactly two distinct shapes are shown (and thus one of them is shown twice) then 1 point is added to the score of the player(s) who showed the winning shape, otherwise no point is added. After several rounds it occurred that each shape had been shown the same number of times. Prove that the total sum of points at this moment was a multiple of 3. *Egor Bakaev*
5	4. In a right-angled triangle ABC ($\angle C = 90°$) points K, L and M are chosen on sides AC, BC and AB respectively so that $AK = BL = a$, $KM = LM = b$ and $\angle KML = 90°$. Prove that $a = b$. *Egor Bakaev*
	5. In a country there are 100 cities. Every two cities are connected by direct flight (in both directions). Each flight costs a positive (not necessarily integer) number of doubloons. The flights in both directions between two given cities are of the same cost. The average cost of a flight is 1 doubloon. A traveller plans to visit any m cities for m flights, starting and ending at his native city (which is one of these m cities). Can the traveller always fulfil his plans given that he can spend at most m doubloons if
3	a) $m = 99$;
3	b) $m = 100$? *Egor Bakaev*

37th INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Senior O-Level Paper, Fall 2015

Grades 10 – 11 (ages 15 and older)

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed up.)

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points	problems
3	1. Let p be a prime number. Determine the number of positive integers n such that pn is a multiple of $p+n$. *Boris Frenkin*
4	2. Suppose that ABC and ABD are right-angled triangles with common hypotenuse AB (D and C are on the same side of line AB). If $AC = BC$ and DK is a bisector of angle ADB, prove that the circumcenter of triangle ACK belongs to line AD. *Egor Bakaev, Alexandr Zimin*
4	3. Three players play the game "rock-paper-scissors". In every round, each player simultaneously shows one of these shapes. Rock beats scissors, scissors beat paper, while paper beats rock. If in a round exactly two distinct shapes are shown (and thus one of them is shown twice) then 1 point is added to the score of the player(s) who showed the winning shape, otherwise no point is added. After several rounds it occurred that each shape had been shown the same number of times. Prove that the total sum of points at this moment was a multiple of 3. *Egor Bakaev*
	4. In a country there are 100 cities. Every two cities are connected by direct flight (in both directions). Each flight costs a positive (not necessarily integer) number of doubloons. The flights in both directions between two given cities are of the same cost. The average cost of a flight is 1 doubloon. A traveller plans to visit any m cities for m flights, starting and ending at his native city (which is one of these m cities). Can the traveller always fulfil his plans given that he can spend at most m doubloons if
2	a) $m = 99$;
2	b) $m = 100$? *Egor Bakaev*
5	5. An infinite increasing arithmetical progression is given. A new sequence is constructed in the following way: its first term is the sum of several first terms of the original sequence, its second term is the sum of several next terms of the original sequence and so on. Is it possible that the new sequence is a geometrical progression? *Georgy Zhukov*