C.7 Let X be the (non-empty) set of $2D$ words that can be formed. We write $c \prec u$ if the finite pattern c appears in $u \in X$, and $u \prec v$ if any finite patterns of $u \in X$ also appears in $v \in X$. A finite pattern c of $u \in X$ is said to be *critical* if there is a sequence (S_n) of patterns of u which cover arbitrarily large disks and such that, for any n, $c \not\prec S_n$.

1. A $2D$ word is quasiperiodic if and only if it has no critical pattern.

2. Assume that $u_0 \in X$ is not quasiperiodic. Let c_0 be a critical pattern of u_0 and (S_n) be the associated sequence of patterns. We use **B.4** to build from (S_n) a $2D$ word $u_1 \in X$. One has $c_0 \not\prec u_1 \prec u_0$.

3. While u_n has a critical pattern c_n, we find as above $u_{n+1} \in X$ such that $c_n \not\prec u_{n+1} \prec u_n$. We moreover take for c_n the smallest one among the critical patterns of u_n. Note that c_n is also critical for u_k for $k \leq n$.

4. If we eventually find $u_n \in X$ without critical pattern, then we are done.

5. Otherwise, we again use **B.4** to build from (u_n) a $2D$ word $u_\infty \in X$. One has, for any n, $u_\infty \prec u_n$. Let us show that u_∞ is quasiperiodic.

6. If u_∞ has a critical pattern c, then c is a critical pattern of any u_n.

7. One has $c_n \notin \{c_0, \ldots, c_{n-1}\}$ because c_n is also critical for u_k, $k \leq n$. The size of c_n is thus not uniformly bounded. For c_n larger than c, this contradicts the minimality of c_n among critical patterns of u_n.